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Risk assessment for synthetic GICs: a quantitative framework for asset–liability management

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Abstract
This study addresses a research gap in quantitative modeling framework and scenario analysis for the risk management of stable value fund wraps, a crucial segment of the U.S. financial market with over USD $400 billion in assets. In this paper, we present an asset–liability model that encompasses an innovative approach to modeling the assets of fixed-income funds coupled with a liability model backed by empirical analysis on a unique data set covering 80% of the stand-alone plan sponsor market, contrasting with models based solely on regular deterministic cash flows and interest rate differences. Our model identifies and analyzes two critical risk scenarios from the insurer's perspective: inflationary and yield spike. Our approach demonstrates that the tail risk of wraps, used as an economic capital measure, is sensitive to characteristic parameters of the fund, such as the duration, portfolio composition and credit quality of assets. This finding significantly differs from U.S. regulatory approaches like the NAIC's, which often result in a zero capital requirement. These findings reveal limitations in current actuarial risk and profitability metrics for U.S. insurers and argue that a more sophisticated risk model reproducing the two critical scenarios is necessary.

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1 Introduction

Stable value funds are an essential investment choice within 401(k) plans, functioning as financial vehicles that ensure capital preservation and augment the range of investment options for investors. The funds' notoriety is highlighted by the third quarter of 2020, where stable value funds managed over USD $888 billion of assets (2020). A study by 2022 observes that over 82% of defined contribution plans provide stable value funds to their participants. The practical utility of stable value funds is also highlighted by Babbel and Herce (2018), who found that can improve the return of a retirement portfolio through better diversification.[1]

Despite their significant role in the U.S. market, academic studies on stable value funds' asset–liability management from an insurer's perspective are scarce, with limited literature such as Kwun et al. (2009) and Kwun et al. (2010). This analytical gap becomes increasingly relevant due to recent trends in stable value funds, where there is a noticeable shift towards lower credit quality and longer-duration assets. This evolution in investment strategy accentuates the need for robust quantitative methods to effectively assess the associated insurance risks.

The 2023 technical defaults of several banks, as highlighted by Dinh (2023), featured distinct participant demographics, especially deposit holders in the venture capital industry, who, as Vo and Le (2023) notes, are likely to know each other, increasing the risk of a bank run. This underscores the importance of understanding the behavioral aspects of participants in asset–liability management. drawing from Alimoradian et al. (2023)'s empirical analysis of 44, 000 monthly cash flows across 297 stable value plans, we model a multicomponent liability model. This model incorporates behavioral factors observed within the participant cash flows such as rate deficit (Dar and Dodds 1989; Sierra Jimenez 2012), reputational mass lapse (Shin 2009), flight-to-safety behavior (Baur and Lucey 2010; Dorn and Huberman 2005), and regime-switching trends.

In this paper, we also introduce a robust asset model that encapsulates the primary risk factors in fixed-income funds. The traditional method for projecting a bond fund in the literature is combining interest rate models with defining fixed coupon payment structure as per Ringer and Tehranchi (2006), Andersson and Lagerås (2013), Alfonsi et al. (2019). However, these models often become overly complex, focusing predominantly on secondary risks such as term-structure basis spread risk. This complexity can overshadow the detection of primary risk factors, namely yield and duration. Our paper aims to address this gap by proposing a more streamlined approach that models the yield, a critical risk factor for diversified fixed-income funds. Utilizing the Cox-

[1] Although some conclusions from this paper may be applicable to stable value funds in general, our primary focus is on individually managed synthetic guaranteed investment contracts (GICs), also known as *wraps* According to 2020, about USD $400 billion is insured via wrap contracts.

Ingersoll-Ross (CIR) process, we simulate the dynamics of spreads and rates affecting the fund's yield, with the calibration of the CIR parameters conducted against historical data using the log-likelihood method.

To complement our approach, we define a script that encapsulates the Synthetic GIC's contractual cash flow terms, enabling us to estimate the insurer's payoffs. Having scripted the three building blocks in a computer language - the asset model scenario generator, the participants' cash flow model scenario generator, and the code for the Synthetic contract's financials, we simulate scenarios using a Monte Carlo framework to assess the Synthetic GIC's tail risk. We also provide sensitivity analysis to various input data.

Based on the simulation scenarios, we conduct scenario detection analysis to identify common patterns in the most significant tail scenarios produced by the Monte Carlo simulations. This scrutiny of significant tail scenarios helps pinpoint the primary risk scenarios for the insurer.[2] This risky scenario detection methodology will further assist us in Sect. 7, to further justify our liability cash flow mode; our liability model's stochastic trend component is key in generating these adverse loss scenarios, in contrast to models like Kwun et al. (2009), which lack an independent stochastic component and struggle to generate high withdrawal rates at critical times. Additionally, the U.S. regulatory approach (2015) for reserve calculation doesn't fully capture the complex, path-dependent nature of insurance risk in this product, often overlooking the riskiest scenarios in cash flow projections

Ermanno Pitacco's seminal work on variable annuities (Bacinello et al. 2011) has profoundly influenced our research. His pioneering approach to modeling insurance-linked products has been a guiding beacon for us, particularly his insightful detection of risk scenarios (Bacinello et al. 2010). Pitacco's blend of theoretical rigor and practical application has not only shaped our research but also continues to influence practices in the insurance industry. The existence of our current research is a testament to the continuity of his innovative thinking.

The rest of the paper is structured as follows: Sect. 2 introduces the stable value ecosystem, elucidates its structural mechanism, and explains the efficiency of such an investment structure. Section 3 and Sect. 4 present the asset model and the participants' lapse liability model, respectively. Section 5 overlays the numerical simulation results while Sect. 6 discusses the risk scenarios for the insurer. Section 7 in particular discusses the path-dependent nature of the risk of this product and the suitability of our cash flow model. The paper concludes with a discussion of the economic capital requirements and best practices in stable value fund guarantee risk management. It provides recommendations for practitioners, rating agencies, and regulators on estimating stable value fund wrap risk in Sect. 8.

[2] Regulatory emphasis on such analysis is highlighted in 2015, recommending reverse stress tests in the ORSA process.

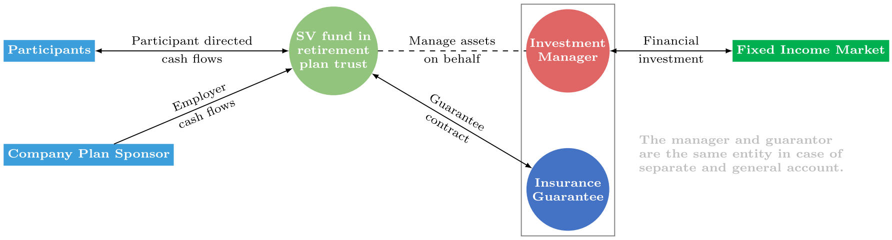

Fig. 1 Stable value ecosystem

2 Stable value guaranteed insurance contract

Stable value funds are investment options offered by many defined contribution retirement plans, generally characterized as very low-risk investment options with liquidity and principal preservation similar to money market funds but with slightly higher returns.[3,4,5]

Figure 1 is a pictorial representation of the stable value ecosystem. Participants, in search of low-risk and principal-preserving investments, make their contributions or distributions to a retirement account managed by a bankruptcy-remote trust. These contributions or distributions are made through an employer or a pool of employers, and an investment entity then invests those assets appropriately.[6] Under the terms of the insurance contract, the assets are guaranteed at book value responsive payment. The details of this book value mechanism will be discussed in the following subsection.

[3] For more details about stable value funds, we refer the reader to the 2020 and Fabozzi (1998) comprehensive handbook on stable value investments.

[4] According to the Stable Value Investment Association's quarterly survey, USD $162 billion of the stable value wrap market is represented by stable value pooled funds, and USD $270 billion are *individually managed* (2020). See (2020) for more details.

[5] Generally speaking, there are three major types of stable value products: *General Account* where the assets of the funds are managed by and invested in the insurance company's general account, *Separate Account* where the insurance company is also the manager of the fund, but the assets of the funds will be segregated from the insurance company's general account, and *Synthetic Guaranteed Investment Contracts* (*Synthetic GICs*) where the insurance company is an independent entity from the manager of the funds (see 2020 for more details). Stable value funds can also either represent one employer or a pool of employers. In the case where the plan sponsor is one employer, the fund is known as *individually managed*. In contrast, pooled funds allow small and medium-sized plan sponsors to bring their stable value funds together in a single fund formed under federal or state banking laws.

[6] Fig. 1 also shows cash flows by company plan sponsor/employer. These cash flows are usually standard contributions, like bonus payments made by the employer. Still, they can also include once-off cash flows due to restructuring after a merger or acquisition, company bankruptcy, layoffs, or other company plan-sponsor corporate actions. The insurance guarantee may have strict contractual terms for these cash flows and may not cover them at *book value payment* under the *wrap agreement* or may cover them up to a certain, predetermined limit. In the case of general and separate account products, the investment and the guarantor roles are performed by the same entity, typically an insurance company. However, with Synthetic Guaranteed Investment Contracts (GICs), the retirement plan trust engages one or more guarantors, typically insurance companies, to ensure a balance between its assets and liabilities.

2.1 Book value wrap accounting

This subsection presents the mathematical formulation for the book value wrap accounting concept in the context of stable value funds. This specific accounting practice allows the fund participants to treat their investments at their original cost plus any credited interest, as opposed to their current market value. This key feature, which allows transactions at book value, provides the principal protection characteristic inherent to stable value funds.

B_t and M_t represent the aggregated book value and market value for the sum of all participant contracts, respectively.

Definition 1 We define the following time-dependent processes for each $t \geq 0$:

- $\{M_t\}_{t\geq 0}$ represents the market value of the fund;
- $\{B_t\}_{t\geq 0}$ denotes the book value of the fund;
- $\{\gamma_t\}_{t\geq 0}$ corresponds to the rate, also known as the crediting rate, that represents the growth of the book value of the fund;
- $\{C_t\}_{t\geq 0}$ represents the cash flow activities in/from the fund.

The book value of the fund evolves according to the following dynamic

$$dB_t = B_t \gamma_t dt + dC_t \, , \tag{1}$$

while we assume crediting $\{\gamma_t\}_{t\geq 0}$ to be described by the following equation:

$$\gamma_t = \left(\frac{1}{\theta_t} \ln \frac{M_t}{B_t} + y_t - p \right)_+ , \tag{2}$$

where $p \in \mathbb{R}^+$ is the insurance premium while $\{\theta_t\}_{t\geq 0}$ (expressed in years) is the duration, and $\{y_t\}_{t\geq 0}$ is the yield of the fund that will be mathematically developed further in this paper.

Remark 1 For general accounts and most separate stable value funds, γ_t can be defined at the discretion of the insurance company. However, for Synthetic GICs and some separate account products, γ_t can be defined by means of an exact formula. While there are several versions for the crediting formulas used within the market, one of the most used formulas corresponds to Eq. (2). Other contracts may use a different crediting rate expression. Still, when expressed in a continuous formulation, they always converge to Eq. (2) (or equivalent have a residual difference w.r.t. this formula).

Note that Eq. (2) is aiming at making the *book value* converge to the *market value* in θ_t years. Hence, for a given yield y_t (resembling the growth speed of the market value M_t) and insurance costs p, the right side of Eq. 2 approximates the crediting rate to achieve the convergence of the book value to the market value in θ_t years.

For example, if we assume no cash inflows/outflows, i.e., if $dC_u = 0$ during the period $[t, t + \theta_t]$, B_t depends only on the crediting rate γ_t. Moreover, if we neglect market risk, i.e., the market value grows at a pace of $y_t - p$, then even if there is an asset–liability mismatch at t, i.e., $B_t \neq M_t$, we desire the two to be the same over θ_t

Table 1 Simplified T-Account from the 401(k) trust's perspective

Assets	Liabilities
Fund's asset (M_t)	Book value (B_t)
Present value of insurance guarantee	Present value of the sum of upcoming insurance premiums

years.[7] However, in reality, both cash inflows and outflows occur, and the yield of the fund changes over time. Furthermore, if $\gamma_t \leq 0$, convergence is no longer guaranteed. This is the motivation behind the fact the fund needs to recourse to an insurance company to cover any losses due to an asset–liability mismatch, which will be further explained in the following Section.

In the context of stable value funds, the retirement plan trust manages assets using an investment entity while being liable for the book value of the assets to the retirement plan participants. To ensure asset–liability parity, the trust enters into a guarantee contract with an insurer.

2.2 Insurance guarantee

Table 1 illustrates a simplified T-account from the 401(k) trust's perspective. On the left side, the trust manages assets using an investment entity. On the right side, the trust is liable for the book value of the assets to 401(k) plan participants. To ensure asset–liability parity, the trust enters into a guarantee contract with an insurance firm and pays insurance premiums. According to the asset–liability matching principle, the book value plus the sum of insurance premiums should always be at par with the assets.

A key characteristic of a stable value fund guarantee is that the insurance company acts as the last resort, meaning that as long as the last individual (or the last pool of individuals) has not withdrawn their assets from the fund, the current participants invested in the stable value fund - and not the insurer - will pay for any past fund shortage. In other words, the insurer will make a claim payment only when the assets of the stable value fund have been fully exhausted. The last resort concept is the core advantage of stable value funds over other guaranteed investment structures, making the insurance guarantee relatively more affordable since, because of the last resorting, the embedded put option becomes very remote and out of the money.

Definition 2 The market value deficit G_t, also known as asset–liability mismatch, is defined as the difference between the book value and market value, i.e., $G_t = B_t - M_t$.

Another key tool is given by the *last resort time*. Given a time horizon T_{max}, the last resort time $\hat{T}$ is defined as the time when the market value reaches below zero $M_{\hat{T}} \leq 0$, thus

[7] In other words, if the right side of Eq. 2 is positive ($\gamma_t > 0$), and if the yield is constant over time, and there are no cash inflows and outflows, ($dC_t = 0$), then the market value and book value will converge in θ_t years.

$$\hat{T} = T_{max} \wedge \inf_{t \geq 0} \{t : M_t \leq 0\} \ .$$

Moreover, $G_{\hat{T}}$ as the insurance loss paid at time $\hat{T}$, the asset–liability mismatch at the last resort when the last participant withdraws, hence the following

$$G_{\hat{T}} = \left(B_{\hat{T}} - M_{\hat{T}}\right)_+ = B_{\hat{T}} \ , \tag{3}$$

holds.

We emphasize that the stochastic resort time $\hat{T}$ corresponds to a random variable. Although the contract theoretically has no fixed maturity, it's common to introduce a time horizon, T_{max}, for practical purposes. This is because parties may have the option to terminate the contract early under legal clauses commonly known as extended termination or wind-down. Also, a defined time horizon is necessary for computational simulation. Here, we note that even though in this paper the stochastic last resort time $\hat{T}$ is determined by computational methods, this variable can also be calculated by means of stochastic optimal control techniques; this analysis is left for further research.

3 Asset model

This section of the paper focuses on modeling the dynamics of the market value of stable value funds, an essential aspect of the asset–liability management framework. Diverging from the methodology of modeling fixed income funds based on the sum of coupons, the model presented in this paper adopts a more generic approach, emphasizing yield as the primary risk factor. This approach is inspired by the way practitioners assess the dynamics and returns of these types of funds.

The following equation describes how the market value of the fund can grow or shrink based on the fund's cash flows (C_t), taking into account the insurance premium (p) as a cash outflow.

$$dM_t = M_t(y_t + \delta)dt - M_t\theta_t dy_t - pB_t dt + dC_t. \tag{4}$$

In the model described by Eq. 4, the yield y_t is crucial as it represents the return on investment. The adjustment rate δ accounts for the increase/decrease in the value of the securities within the fund due to factors such as the convexity of the fund, impaired securities, operational volatility, etc. Therefore, in our model, the net expected growth rate of the fund is captured by $y_t + \delta$. In the real world, the parameter δ can be dynamic and stochastic, possibly influenced by various factors including the yield. However, for simplicity in our model, we assume δ to be constant. It's also worth noting that a variation of the model, as seen in Eq. 4, is discussed in the works of Jarrow (1978) and Hopewell and Kaufman (2017).

By concentrating on the yield as the primary risk factor and incorporating the effective duration, this model provides a more suitable approach for understanding and managing the risks associated with fixed-income funds.

To complete our model, we have to define the dynamics of the yield $\{y_t\}_{t\geq 0}$. A natural approach to modeling the yield of a fixed income fund involves decomposing it into the sum of a risk-free rate r_t and a spread s_t given by:

$$y_t = r_t + s_t \,. \tag{5}$$

The spread s_t is commonly known as an option-adjusted spread, or *OAS*.[8] We also assume that the interest rates and OAS evolves according to the following Cox-Ingersoll-Ross ("CIR") model.

$$\begin{aligned} dr_t &= \kappa_r(r_t - r_\infty)dt + \nu_r\sqrt{r_t}dW_t^r \\ ds_t &= \kappa_s(s_t - s_\infty)dt + \nu_s\sqrt{s_t}dW_t^s \\ \langle W_t^s, W_t^r\rangle &= \rho t. \end{aligned} \tag{6}$$

The parameters of the Cox-Ingersoll-Ross (CIR) model are detailed as follows: The mean reversion levels, r_∞ for rates and s_∞ for spreads, represent the long-term average levels. These parameters are the asymptotic values of the rate r_t and spread s_t as t becomes large. The speed of mean reversion is defined by κ_r for rates and κ_s for spreads, indicating how rapidly the reversion towards the mean occurs. The volatilities, ν_r and ν_s, measure the extent of fluctuation in rates and spreads. The model also specifies the initial values for rates and spreads. The Brownian motions W_t^s and W_t^r introduce randomness into the model, with ρ representing the correlation between these two stochastic processes.

4 Liability cash flow model

In this section, we present a model for the liability cash flow dynamics of the stable value fund, focusing on the cash flows generated by the participants' contributions and withdrawals. Recall that C_t is the stochastic process representing the accumulated cash flows by the participants of the stable value fund. A positive C_t indicates the growth of the stable value fund, while a negative C_t signifies withdrawals from the fund. We define the dynamics of $\{C_t\}_{t\geq 0}$ as per Eq. 7:

$$dC_t = B_t\left(\underbrace{\beta_t}_{\text{structural trend including herd behavior}} + \underbrace{g(r_t - \gamma_t)}_{\text{rate deficit}} + \underbrace{f(s_t)}_{\text{flight-to-safety}}\right)dt. \tag{7}$$

As it can be deciphered from Eq. 7, the model is composed of key components whose relevance to liability cash flows will be elaborated upon in the rest of this section.

- A structural, non-monotonic regime-changing trend in cash flows related to the nature of plan sponsors' ecosystem, which indirectly influences participants' behavior. In our model, this is captured by the term β_t.

[8] see Choudhry and Lizzio (2015); Elton et al. (2001).

- A rate deficit risk factor: This is represented in our model by the term $g(r_t - \gamma_t)$. The rate deficit hypothesis, as illustrated in Alimoradian et al. (2023); Barsotti et al. (2016), suggests that withdrawals from financial products can be modeled using interest rate differentials between the product in question and other Conflict of interest rates.
- A herd behavior component: This component is also included in our model's β_t term.
- The cash flow risk-mitigating effect of flight-to-safety behavior during a crisis: In our model, this is captured by the term $f(s_t)$.

4.1 Regime changing trend component

The regime-changing trend component is important for capturing the dynamics of different market conditions and plan-sponsor ecosystem and their impact on the participant's cash flows. Alimoradian et al. (2023) conducted empirical data analysis and identified non-monotonic trends within the historical participant cash flow data; they observed that these trends are stochastic, varying in both size and duration, with notable change points. Importantly, they find no strong correlation between trends before and after a change point. This lack of correlation is attributed to an economic intuition that these trends are indicative of the prevailing market environment and the overall health of the plan-sponsor ecosystem.[9] These observations align with the findings of Madrian and Shea (2001) and Mitchell et al. (2006), who analyzed individual 401(k) account data and concluded that the plan-sponsor's role is critical in investment allocation decisions. They found that approximately 10% of liability cash flow activities by participants of the fund were found to be influenced by the plan sponsor. Notably, these changes in trends are often structural and unpredictable, contingent on factors like management decisions that extend beyond quantifiable economic variables. For instance, a company with a history of stable employment may continue this trend in the subsequent years. However, if there is a regime change, the trend could swing in any direction.

We define the set $\{S_i, \tau_i\}$ as the i-th state and its time of switching to this state. With this notation, Eq. 8 relates the trends β with the states. In addition, the duration of each state $(\tau_{i+1} - \tau_i)$ is stochastic, and we assume that it follows an exponential distribution with the intensity parameter $\frac{1}{d_i}$. Note that d_i does have a simple economic interpretation; it is the average duration in years of being in the regime S_i. Therefore, we have:

$$\begin{aligned} \forall \tau_i \leq t \leq \tau_{i+1}, \quad \beta_t &= b_i \\ P(\tau_{i+1} - \tau_i \geq x) &= e^{-\frac{x}{d_i}} \\ P(S_i = s) &= p_s . \end{aligned} \tag{8}$$

[9] Factors such as plan sponsor communication and management decisions, financial health, industry sector, employment policies, growth or layoffs, plan demographics, and default options can all impact participant cash flow trends.

Table 2 Participant trends probability of occurrence and their average duration

State	Respective trend	Probability of occurrence	Average duration
Stable state	b_0	$1 - p_1 - p_2 - p_3(\frac{M_t}{B_t})$	d_0 years
Decline state	b_1	p_1	d_1 years
Growth state	b_2	p_2	d_2 years
Herd state	b_3	$p_3(\frac{M_t}{B_t})$	d_3 years

As discussed earlier, in Eq. 9, we assume that the trend of the next regime, β_{i+1}, is independent of the trend of the current regime, β_i.

$$(\beta_{i+1}, \tau_{i+1}) \perp\!\!\!\perp (\beta_i, \tau_i), \tag{9}$$

In other words, we assume the parameters of the next regime do not depend on those of the current regime.

To round out our model, we discretize the state process into four states: growth, stability, and decline, and we include an additional herd state. This is depicted in Table 2. The growth state signifies a positive cash flow trend for the fund, whereas the decline state signifies a trend of withdrawals. Note that in Table 2, the probability of occurrence of the herd state is a function of the market-to-book value ratio, while the probabilities of occurrence of other states are considered constant.

Table 2 presents a regime-switching model with three main states and an additional state to account for herd behavior. A three-state model offers the benefit of alternating between high, stable, and low withdrawal rates at random intervals. It is worth noting that the goal here is not to prescribe the exact number of states needed for cash flow projections. Instead, we aim to provide a generic model for liability cash flows that aligns with behavioral studies, as seen in Alimoradian et al. (2023). Therefore, an insurance industry practitioner could choose to apply to fewer or more states based on the specific experience of the plan.

4.2 Herd component

The herd state is an important component in our model because it accounts for the potential impact of mass withdrawals on the insurer's liability. This behavioral component was observed in stable values by Alimoradian et al. (2023). Further, according to studies by Loisel and Milhaud (2011) and Barsotti et al. (2016), peer-to-peer influence and contagion effects can lead to mass correlation in withdrawals among policyholders' cash flows, resulting in mass lapses. Alimoradian et al. (2023) also note that a low market-to-book value could potentially be perceived as a weakness and hypothetically lead to reputational issues, increasing the likelihood of herd behavior. Therefore, in Table 13, the herd behavior is shown in the last trend component $p(\frac{M_t}{B_t})$ as a function of market-to-book value.

4.3 Rate deficit component

Equation 7 also has a rate deficit component shown by the function $g(\cdot)$ as the difference between the yield of the fund and the crediting rate of the stable value fund. This concept, a well-recognized hypothesis in literature, has been empirically examined in studies such as Outreville (1990), Tsai et al. (2002), Floryszczak et al. (2016), Barucci et al. (2020). In Alimoradian et al. (2023), the authors analyze historical data for the years 2000 to 2021 to observe any relationship between the participant's lapses and the interest rates. However, the authors did not observe any relationship between the rate deficits and the cash flow rates. Still, they pointed out that the historical period of 2000 to 2021 was limited as the market was not exposed to an inflationary period where interest rates rose significantly.

4.4 Flight-to-safety component

The flight-to-safety hypothesis, as discussed in Baur and Lucey (2010), Ji et al. (2020) and Dorn and Huberman (2005), posits that during times of market stress, investors tend to shift their investments from riskier assets to safer assets. The risk-mitigating effect of flight-to-safety was empirically observed in stable values by the study of Alimoradian et al. (2023). This factor should thus be considered when modeling stable value fund liability cash flows to account for potential shifts in participant behavior during market crises.

In this work, we model the 'flight-to-safety' effect using the function $f(s_t)$, where s_t represents the OAS spread of the fund. One might question why we have chosen s_t as an indicator of market distress and the flight to safety rather than other indicators, such as equity performance. We have two primary reasons for this choice.

First, stable value funds are diversified investment portfolios encompassing a large portion of U.S. fixed income markets. Therefore, these funds are highly correlated with and representative of the U.S. fixed income market. A common indicator of the U.S. financial market entering a crisis is a widening global credit spread, as discussed in Salhi and Thérond (2014). Given its diversified nature, a stable value fund would also experience this widening of spreads during a crisis.

Secondly, the OAS spread is an internal risk factor to the system, impacting both assets and liabilities. Using an internal risk factor is more coherent with the model structure and ensures that the flight-to-safety component is organically integrated into the system. If we chose an external factor, it would be equivalent to adding an independent stochastic component to the model. For instance, if equity performance was used as the argument to the function $f(.)$, it would only be material to the risk measurement if there is a significant correlation between interest rates and equities. Assuming no strong correlation exists, incorporating equities would merely add an independent stochastic factor that would be more appropriately introduced directly into the stochastic trend component as an extra trend state representing process.

Table 3 Loss frequency for different probabilities of entering a declining state in a year, and different declining state annual withdrawal rate

	$p_1 = 1/100$	$p_1 = 1/50$	$p_1 = 1/20$	$p_1 = 1/10$
$b_1 = 1/5$	0.08%	0.12%	0.23%	0.45%
$b_1 = 1/4$	0.09%	0.15%	0.39%	0.84%
$b_1 = 1/3$	0.19%	0.36%	0.86%	1.80%

Table 4 99% CTE of present value (PV) of loss, comparison for the different probabilities of entering a declining state in a year, and different declining state annual withdrawal rate

	$p_1 = 1/100$	$p_1 = 1/50$	$p_1 = 1/20$	$p_1 = 1/10$
$b_1 = 1/5$	0.05%	0.20%	0.26%	0.50%
$b_1 = 1/4$	0.06%	0.28%	0.53%	0.85%
$b_1 = 1/3$	0.14%	0.53%	1.24%	2.24%

5 Wrap risk results

This section outlines the main results from the Monte Carlo simulation of the model we described earlier. Before diving into the specifics, we will introduce an added assumption to the model. In our model framework, the effective duration, represented by θ_t, indicates how sensitive an asset is to changes in its yield. For the discussions in this paper, we will assume that the effective duration remains constant. This is a reasonable assumption because stable-value funds often have fixed durations due to the investment constraints set by their respective insurance companies. So, for the rest of this paper, we assume that $\forall t \geq 0 \quad \theta_t = \theta$.

Table 3 shows the loss frequency under different parameter assumptions for the probability of transitioning into a decline (p_1) and different definitions of a decline (b_1). Other parameter inputs are as described in Appendix A. Table 3 also shows the convexity of the wrap risk resulting when the loss frequency is low, an indication of the remoteness of the risk. [10]

In our analysis presented in Tables 4 and 3, we set the correlation parameter (ρ) between interest rates and spread brownians as zero. Recognizing the sensitivity of our results to this parameter, we conducted an expanded analysis across different correlation levels, as shown in Table 5. Table 5 illustrates that a higher correlation between OAS and interest rates slightly heightens the tail risk for insurance guarantees. This is intuitively expected, as higher correlation between the two variables, implies higher volatility for the fund which means higher chances of the fund reaching to lower market-to-book value ratios. However, the Table 5 also shows that the results are not very sensitive to this parameter.

As detailed by Duffee (1998), the relationship between interest rates and OAS spreads is complex and regime-dependent, often exhibiting a negative correlation. In

[10] Unless otherwise stated, Portfolio 1 from Table 6 is used by default, $\rho = 0$, and $\delta = -0.07\%$, for all risk metrics analyses conducted.

Table 5 Loss frequency and 99% CTE of PV of loss for different correlation between rate and spreads' Brownians.

	$\rho = -0.5$	$\rho = 0$	$\rho = 0.5$
Loss frequency	0.22%	0.23%	0.24%
99% CTE of PV of loss	0.25%	0.26%	0.28%

This analysis assumes a probability of transitioning into a decline state ($p_1 = 1/20$) and defines a decline ($b_1 = 1/5$)

addition to Duffee (1998), which focused on corporate bonds, one may realize by looking at Table 6 that there is a very high concentration on mortgage-backed securities in stable value funds. These securities are known to experience duration convexity, and consequent OAS spread variations due to prepayment behaviors during rate fluctuations (Hanson 2014; Boudoukh et al. 1997). This adds another layer to the nuanced interplay between interest rates and OAS spreads for stable value funds. However, our analysis in Table 5 indicates that the first-order risk sensitivity to the correlation factor ρ is not substantial. Therefore, even though the relationship is more complex than just correlating two Brownians, our choice of the model is fit for the purpose simply because the sensitivity of the tail risk of the guarantee to this relationship seems to be limited.[11] Given the non-stationarity and predominantly negative correlation, along with the limited sensitivity to ρ, we maintain a zero correlation assumption for the remainder of this paper.

Table 6 shows that stable value funds consist of various asset classes, including Asset-Backed Securities (ABS), Residential Mortgage-Backed Securities (RMBS), Commercial Mortgage-Backed Securities (CMBS), and Mortgage-Backed Securities (MBS). This table presents three portfolios with varying risk levels based on their allocations to these securities: Portfolio 1 has the lowest risk, Portfolio 3 the highest, and Portfolio 2 represents a medium risk profile. We, therefore, analyze the impact of credit quality and asset composition on wrap risk.[12] We then performed a risk analysis for these different portfolios in Table 7 shows the risk profile of the wrap for Portfolio 1, 2, and 3; it shows how the stable value wraps tail risk increases when guaranteeing riskier assets.

Most stable value wraps do not cover, or only partially cover, the default risk of the assets. However, in practice, investment managers can sell the assets before any default occurs. Therefore, the fund will still likely experience a market value loss due to credit rating migration. This credit migration risk is embedded in the parameter δ of Eq. 4 of our model.[13] We note that, in the real world, δ can be time-dependent and stochastic

[11] Insurers, however, may opt for more sophisticated models, or predefined interest rates and bond spread scenarios within their enterprise risk management framework, or provided by regulatory and policymaking bodies (Actuaries 2005, 2020).

[12] In order to do so for each of the portfolios described in Table 6, the OAS CIR parameters are calibrated to historical time series as detailed in Appendix B.

[13] One way to obtain an estimate of δ is to use the transition probability matrix as discussed in Caouette et al. (1998), if we calibrate this to the tables from Caouette et al. (1998), for example, for AA-rated investments corporate bond, we obtain $\delta = -0.07\%$[14] However, as it can be deciphered from Table 6, the composition of a stable value fixed income fund is more diverse, and the migration risk for the asset classes within the fund has possibly a different probability transition matrix of a downgrade than corporate securities discussed in Caouette et al. (1998).

Table 6 Asset Allocation for three investment portfolios. Portfolio 1 is of lowest risk profile, followed by Portfolio 2 and 3 which are respectively of higher risk

Sector	Portfolio 1	Portfolio 2	Portfolio 3
ABS	15%	17.5%	15%
A-rated ABS	0%	5%	10%
Agency MBS	15%	10%	10%
Non-Agency CMBS	10%	17.5%	20%
Mezz CMBS	0%	5%	5%
AA-rated CMBS	0%	5%	5%
A-rated CMBS	0%	0%	10%
Corporates	30%	30%	35%
High Yield	0%	5%	15%
Non-Agency RMBS	0%	5%	10%
Governmental Securities	30%	20%	10%

Table 7 Guarantee risk of funds with different investment portfolios for a wrap contract specified in Sect. A.

	δ Assumption	99% CTE of PV of loss	Avg. loss size	Loss Frequency
Portfolio 1	−0.05%	0.53%	2.42%	0.36%
	−0.10%	0.55%	2.14%	0.43%
Portfolio 2	−0.20%	0.57%	2.34%	0.41%
	−0.30%	0.64%	2.17%	0.50%
Portfolio 3	−0.50%	0.72%	2.58%	0.46%
	−0.70%	0.89%	2.28%	0.68%

For this analysis, we assumed $b_1 = 1/4$, and $b_1 = 1/20$

in nature. However, for simplicity, we assume that δ is constant. Nevertheless, given this simplification, we consider a conservative approach in setting this parameter, and also for completeness, we provided results for a range of values for the δ parameter in Table 7.

To ensure our paper's findings are not overly dependent on the time of publication, we did not set the initial OAS spread (s_0) to the most current market-calibrated level in any of the results shown in this paper. Instead, we used the long-term mean level ($s_{\inf}$) for s_0. However, to study how changes in the current OAS spread level might affect our results, we conducted additional analyses. The outcomes of these analyses are presented in Table 8, demonstrating that the tail risk of wrap has little sensitivity to variations in s_0.

One factor that has a strong impact on the tail risk of the guarantee is the duration. Table 9 shows that funds with higher duration have significantly greater tail risk. As an example, the loss frequency increases from 0.03% to 3.22% from a 3-year to a 6-year stable value fund duration.

Table 8 Loss frequency and 99% Conditional Tail Expectation (CTE) of the present value (PV) of loss for different initial OAS spread levels (s_0) of Portfolio 1.

	$s_0 = s_\infty - 0.50\%$	$s_0 = s_\infty$	$s_\infty + 0.50\%$
Loss frequency	0.41%	0.39%	0.37%
99% CTE of PV of loss	0.54%	0.53%	0.53%

In this analysis, the long-term mean level of the OAS spread (s_∞) is kept constant, equivalent to the value calibrated in Table 12. Additionally, the analysis assumes a probability of transitioning into a decline state ($p_1 = 1/20$) and defines a decline ($b_1 = 1/4$)

Table 9 Guarantee risk for different asset durations. For this analysis, we assumed $b_1 = 1/5$, and $p_1 = 1/20$

Duration in years	99% CTE of PV of loss	Loss frequency	Avg. loss size
3	0.01%	0.03%	1.16%
4	0.26%	0.23%	2.05%
5	1.53%	1.55%	1.99%
6	5.35%	3.22%	3.42%

Lastly, we also reviewed the statutory reserve and capital requirements for the wrap contract and compared it with the tail risk of the model presented here.

The U.S. National Association of Insurance Commissioners (*NAIC*) model regulation for *reserve* estimation for *Synthetic GICs* are described in (2015). The execution of the model regulation, generally requires insurers to project liability cash flows, presupposing a minimum guaranteed rate, typically set at 0% for *individually managed* Synthetic GICs. Since liabilities almost invariably possess a 0% crediting rate floor, the present value of liabilities predominantly falls below the market value of assets. Therefore, for a typical *individually managed* Synthetic GICs contract similar to the one presented in Appendix A, the reported reserve is almost always 0, and the reserve margin is positive. The reserve margin is defined as the positive part of the difference between the market value of assets and the present value of projected guaranteed liabilities.

Also, NAIC's Risk Based Capital (RBC) for Synthetic GICs are detailed in 2021.[15] The NAIC risk-based capital rules treat Synthetic GICs akin to guaranteed separate accounts, as deduced from 2021:

> "Synthetic GICs are contracts with provisions similar to separate accounts with guarantees, except that the insurance company does not own the assets. For businesses of this type, the C1 and C-3 risk-based capital is determined to be

[15] Since the emergence of the principles-based approach, especially *Solvency I* and *II* by the European regulators, NAIC has also adopted a *principles-based approach*, which applies stochastic modeling under *C3 Phases* (2008). This approach corroborates with the International Association of Insurance Supervisors (*IAIS*) initiative to develop an International Capital Standard (*ICS*) and a comparable measure of capital across different jurisdictions.Currently, large global insurance groups are subject to different capital standards that make it difficult to compare their solvency positions across jurisdictions. The main objectives of the ICS are to establish a common language for supervisors to discuss the solvency of IAIGs and to enhance global convergence among the group capital standards that are in place.

Table 10 Monte Carlo based estimation of the stable value fund wrap risk. For this analysis, we assumed $b_1 = 1/5$, and $p_1 = 1/20$.

NAIC A-695 Reserve	NAIC A-695 Capital	99% CTE of PV of loss-premium	Loss frequency	Avg. loss size
0%	0%	0.26%	0.23%	2.05%

Here, we considered a typical stable value guarantee contract specified in Appendix A

> the same as if the insurance company owned the assets and provided the same guarantees as in a guaranteed separate account."

Further elaboration on guaranteed separate accounts interprets C-1 and C-3 components, with potential offsets by the reserve margin. 2021 statement:

> "Non-Indexed Separate Accounts: … For contracts valued using the fair value of assets and the fair value (at current interest rates) of liabilities, risk-based Capital is calculated as the excess of the regular C-1 and C-3 standards over the applicable reserve margins…"

It is worth noting that the Appendix for RBC capital in Fabozzi (1998) does not explicitly mention this offsetting effect. However, the interpretation posits that any computed C-1 and C-3 capital can be offset against any reserve margins. Given that a typical individually managed Synthetic GIC, such as the one presented in Appendix A, possesses a positive reserve margin, effectively neutralizing the C-1 and C-3 capital impact. Therefore, due to this potential offset of both C-1 and C-3 risks by the reserve margin, risk-based capital typically culminates in zero allocation.

Table 10 illustrates the NAIC reserve and capital charges for a U.S. insurer under the formulaic approach and the assumption of reserve estimates regulation 2015. For completeness, we also assessed the 99% *conditional tail expectation* (*CTE*), the loss frequency, and the average loss size.

6 Risk scenarios to be considered by an insurer

The main objective of this paper is to introduce the readers to the risk scenarios from the insurer's perspective. In order to do this, we projected assets and liabilities within a Monte Carlo framework and studied the scenarios in which losses are observed. This exercise helped us to identify two common patterns: the *yield spike mean reverting scenario* and the *inflationary scenario*. In this section, we discuss these scenarios and provide historical backtests to assess their validity.

6.1 Inflation

Figure 2 shows the average yield and withdrawal rates for the loss threshold of 0.1% centile. This is the highest threshold of losses, and, as can be seen, the most extreme

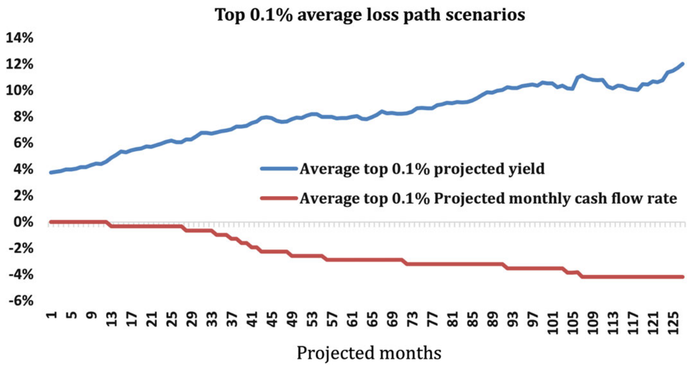

Fig. 2 Top 0.1% average loss path scenarios. Data source: Monte Carlo simulation

scenarios occur when the yields are rising with strong withdrawal rates occurring simultaneously. We call these scenarios the *inflationary scenarios*.

Inflationary yield rises have already happened historically, specifically during the inflationary period of the late 1970 s/early 1980 s and during the Great Depression Period of the 1930 s. Although stable value funds did not exist at the time, evaluating a hypothetical wrap risk during these periods may provide insight into the magnitude of the risk. Therefore, without any explicit assumptions, we back-test a wrap contract against these periods.

1970 s Inflationary Period: In the late 1970 s and into the early 1980 s, there was an inflationary period that experienced sustained double-digit interest rates for approximately three years (Fig. 3). From the perspective of stable value fund assets, this scenario would have exhibited a longer period of adverse market conditions than the 2008 financial crisis. However, higher yields also allow the crediting rate reset mechanism to potentially converge to differences between market values and book values. Figure 4 below shows a hypothetical loss scenario over this period. Although stable value funds did not exist at the time, we used historical data from the Federal Reserve to construct this hypothetical scenario. It shows an insurance loss scenario with cash withdrawal rates of 40% annually ($\sim$ 4.16% monthly) for a sustained period of 7 years. In the figure, the fund yield is the historical yield level obtained by summing the 3y UST rates and the OAS spreads. Interest rates were based on 2020, and OAS spreads were estimated using US Moody's equivalent Baa corporate bonds available from 2020.

Great Depression Period: Interest rates also reached double digits during the *Great Depression*, starting in the late 1920 s. With the Great Depression lasting approximately ten years, the adverse conditions would have lasted much longer than the 2008 financial crisis. Figure 4 shows a hypothetical loss scenario over this period. The graph shows the losses incurred for a 60% annualized withdrawal rate for a sustained period of 3 years. Once again, high yields combined with worsening withdrawal rates produce an environment that can result in sustained losses.

COVID-19 Pandemic: Post-COVID-19 pandemic is not yet known, the U.S. economy has experienced high inflation and rising interest rates during much of the

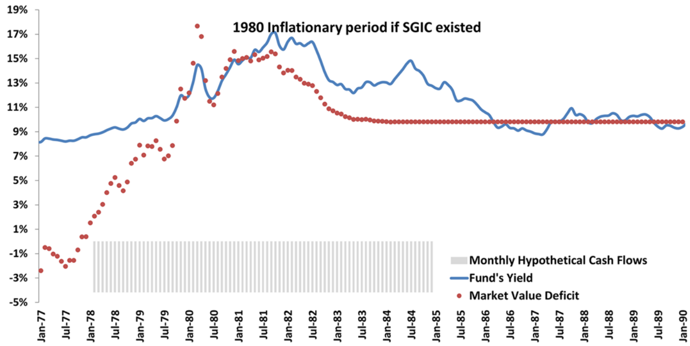

Fig. 3 Hypothetical Synthetic GIC contract with losses during the Inflationary Period if stable value funds had existed at the time. Data source: Simulated backtest against historical data by 2020

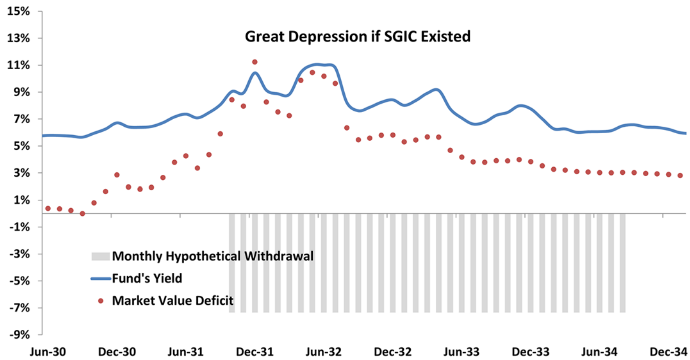

Fig. 4 Great Depression hypothetical scenario. The same data sources as the inflationary period were used for this analysis. Data source: Simulated backtest against historical data by 2020

aftermath of the pandemic with stable value funds, consequently experiencing asset–liability mismatches that could result in a loss when coupled with a high withdrawal rates. Figure 5 shows the consequence of the aftermath of the pandemic period for a very high withdrawal scenario on a hypothetical fund. The long-term effect of the pandemic is not yet known, but for the example wrap contract, there is an 11% market value to book value deficit, so the wrap may incur losses if it experiences a high withdrawal rate in the future.

6.2 Yield spike

Figure 6 shows another pattern observed within the 1% loss scenarios. The pattern shows a period in which yields rise and withdrawals co-occur, followed by sustained lower yields, which we refer to as *yield spike mean reverting scenarios*. The *mean reverting scenario* is when yields rise sharply and then subsequently revert back to more normalized levels. While the normalization of market conditions would likely suggest the mitigation of risk, this scenario is risky if it coincides with very large withdrawals during high yield periods. Suppose large withdrawals do not occur during

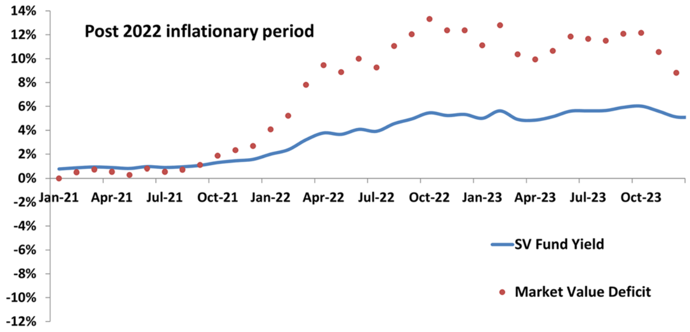

Fig. 5 Post COVID-19 pandemic inflationary scenario assuming no liability cash flows (positive or negative). Data source: Valerian Capital Group LLC proprietary data for the SV Fund Yield

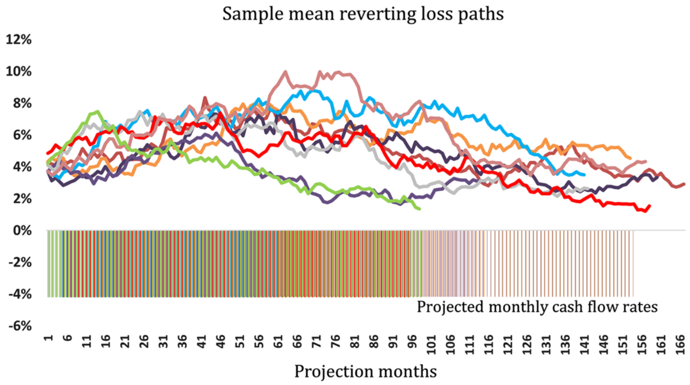

Fig. 6 The *yield spike mean reverting scenarios* observed within the 1% of loss scenarios. Data source: simulated single scenario

high yield periods. In that case, mean reversion will have a lesser impact since there will be capital gains that offset any capital losses that occurred during the original increase in yields. As such, the path to a loss is a "run" on the stable value fund with a large proportion of the book value balances withdrawn over a relatively short period of time. The interrelationship in this scenario is path dependent; higher yields followed by mean reversion and withdrawals combine to create a low-yielding environment without a likelihood of market-to-book value convergence.

Financial Crisis: The reader may again realize that mean-reverting scenarios have also happened historically, specifically during the 2008 financial crisis. During the financial crisis, there were adverse conditions that reduced the market value of stable value fund assets. However, most of the funds did not experience net cash outflows, so there were no actual wrap contract losses experienced during the financial crisis.[16] Furthermore, as shown below in Fig. 7, the market value of stable value fund assets rose significantly by mid-2009 as shown by the negative market value deficit, i.e., by

[16] Source: Aggregated cash flow index from Valerian Capital Group.

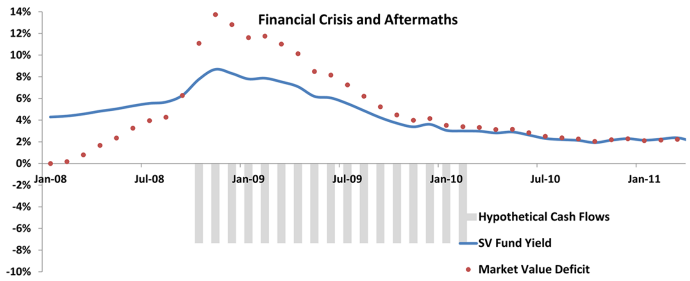

Fig. 7 Withdrawal scenario that would have generated losses as a result of the financial crisis. Simulated backtest

mid-2009, the market value had returned to a level equal to or greater than the book value.

Figure 7 shows a loss scenario under a potential cash flow withdrawal rate. The graph shows a loss scenario generated by a 60% ($\sim 7.35\%$ monthly) annualized withdrawal rate occurring for a sustained period of 18 months.

The *inflation* and *mean reverting yield spike* are the two riskiest scenarios observed within our projected paths. In both of these scenarios, a common characteristic is that an increase in yields aligns with significant cash flow withdrawals. When yields rise, the market value of assets, as well as the gap between assets and liabilities, expands. If this coincides with substantial withdrawals, the gap becomes irrecoverable, leading to losses for the insurer. Another way to describe the economics behind the above scenarios is through the time of recovery of an asset–liability mismatch. In the discussion in Sect. 2.1, we showed that if there is no market risk, and no cash flow risk, and the crediting rate formula has not reached its plateau of 0%, then book and market value will converge over θ years. This last condition in our previous statement is crucial. If the market-to-book ratio is very low, one can see that crediting rate formula 2 will result in a 0% crediting rate and therefore, the recovery time will be much longer. In addition, when we have strong withdrawals coinciding with a low market-to-book value, each withdrawal will worsen the market-to-book ratio hence in mathematical terms we have: $\dfrac{M_t}{B_t} < 1 \Rightarrow \dfrac{M_t - \Delta}{B_t - \Delta} < \dfrac{M_t}{B_t}$. Therefore, the riskiest scenarios from the insurance perspective are withdrawals occurring during low market-to-book ratio, because they will make the convergence of market-to-book to par longer, if not impossible. Such a scenario of low market-to-book ratio as such can occur only if the yields rises. This is the basis of the *inflation* and *mean reverting yield spike*.

7 Path dependence of synthetic GIC Risk and the suitability of the liability model

This section investigates the path-dependent nature of Synthetic GIC risk and our model's aptitude for generating scenarios that replicate the riskiest paths.

In section 6, we identified inflation and mean-reverting yield spikes as the primary risk scenarios. When these scenarios coincide with large withdrawals, they lead to

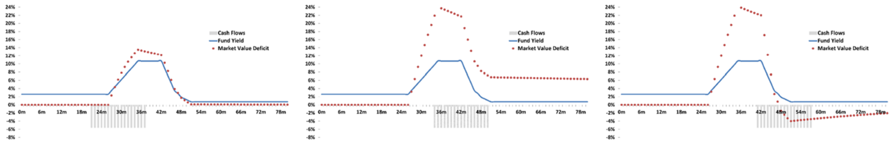

Fig. 8 Three variations of the mean reverting scenario. Data source: simulated backtest.Data source: simulated single scenario

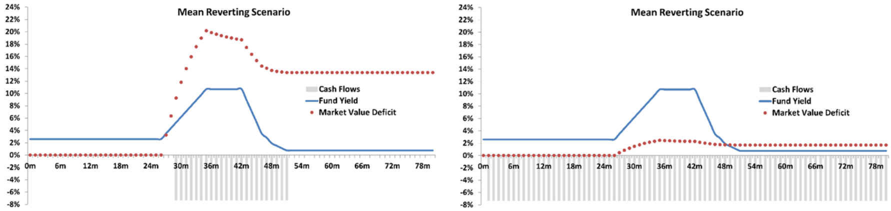

Fig. 9 A mean-reverting scenario example. If a deterministic model were used instead, the loss size would be less than 2% of the initial book value. Conversely, combining cash flow withdrawals with the stable value fund's significant market value decrease results in an approximately 11% loss of the initial book value. Data source: simulated backtest

insurer losses. However, the timing of withdrawals is crucial-they must follow the yield increase, but not too distantly.

Figure 8 presents three variations of the mean-reverting scenario. The left graph shows a scenario with early withdrawals, and the right graph depicts late withdrawals. Neither of these scenarios results in losses. Conversely, the central scenario represents a substantial loss payment, demonstrating the significant impact of withdrawal timing on insurance loss.

Indeed, not only is the timing of withdrawals crucial but so too are the periods without withdrawals. Figure 9 compares the same mean reverting scenario with a deterministic model with constant withdrawal rates and one where the same withdrawal rates only occur during a yield spike. A severe loss of about 11% of the original notional results is evident when the withdrawal rate occurs within 24 months (left graph). Conversely, if a deterministic trend model were used, the loss would be just below 2% (right graph).

Figure 10 further emphasizes the impact of synchronizing the yield spike period with the withdrawal period by comparing the insurance loss size for various withdrawal starting dates. The same yield spike scenario forms the basis for this comparison, with annual withdrawal rates of 20%, 40%, and 60% over 48 months. The x-axis marks the withdrawal starting period relative to the yield spike initiation date. Negative values on the x-axis denote withdrawals starting before the yield increase, and positive values indicate withdrawals beginning after the yield increase. The most substantial insurance loss arises when the withdrawal kick-off aligns with the yield spike onset.

The stochastic state trend component of our cash flow model provides flexibility in high withdrawal rate durations and starting periods, as well as non-withdrawal periods. Therefore, our model can generate the riskiest paths due to this stochastic trend component. However, if our model only incorporated a rate-deficit component,

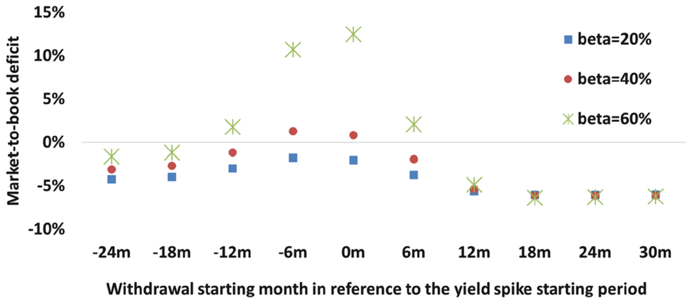

Fig. 10 Insurance loss size (Market-to-Book value deficit) as a function of the withdrawal starting date relative to the yield spike initiation date. Data source: simulated backtest

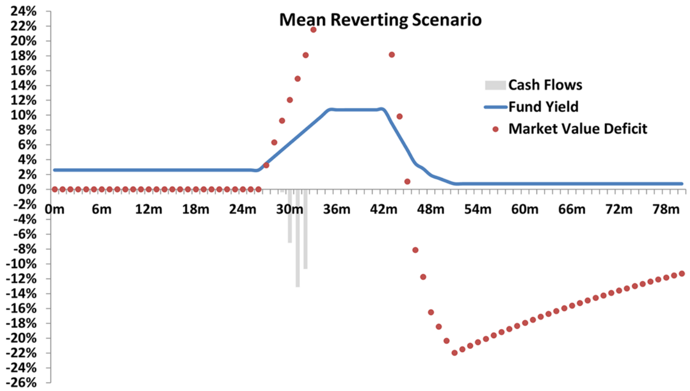

Fig. 11 Mean reverting scenario with a linear rate-deficit withdrawal model $g(.)$. Data source: simulated backtest. Data source: simulated backtest

akin to the model in Kwun et al. (2010), the mean-reverting scenario depicted in Fig. 11 would generate no losses as withdrawals would not be correctly timed.

We should note that while the examples given primarily focused on the *mean-reverting yield spike scenario*, we also tested the *inflationary scenario*. Our findings confirm that for both scenarios, the highest insurance loss occurs when periods of high withdrawals coincide with the yield spike or inflationary period.

In conclusion, our cash flow model, with its stochastic multi-state *beta* component, proves an adaptable tool for the valuation and risk management of insurance liabilities arising from stable value wrap contracts. The path-dependent nature of Synthetic GIC risk necessitates a model capable of generating a wide variety of loss scenarios with variable withdrawal timings and durations. Our model's flexibility, combined with its grounding in empirical participant behavior, allows it to generate nuanced risk scenarios critical for effective risk management of stable value funds.

8 Conclusions

The paper describes a methodology for insurance companies to use within their ERM framework to better understand risk scenarios, assess tail risk, and determine capital requirements for stable value guarantees. Hence, our model can be used to estimate the capital requirements for the insurance company to maintain a certain low probability of insufficient capital for loss payments. In this research, we proposed and tested an asset and liability model for stable value fund wraps.

On the asset side, we suggested an innovative approach to modeling fixed income funds that offers a compelling alternative to traditional arbitrage-free interest rate models, which we argue are ill-suited to handle large portfolios of fixed-income funds. Our model's core strength lies in its ability to capture the primary risk factor for fixed-income funds, the yield, despite its simplicity.

On the liability side, we introduced a multi-component model encapsulating a rate deficit, regime switching trends, herd behavior, and a flight-to-safety component. We argue this is the most suitable model for detecting risk scenarios from the insurer's perspective. Furthermore, our model is consistent with empirical observations that detect long-to-medium-term trends within historical data.

Through our research, we identified two critical scenarios: the inflationary scenario and the mean reverting yield spike scenario. The importance of these scenarios lies in their ability to provide comparative notes on different risk sectors, aiding companies in assessing their aggregate risk exposures.

Economic capital requirements were a focus of our research. Our findings showed that the tail CTE, as a measure of economic capital, is starkly different from the NAIC's approach, which almost always results in zero capital. We noted the significance of understanding the differing statutory capital requirements under various regulations. While we did not explicitly calculate the Solvency II capital for a typical fund, our analysis suggests that it would likely be near zero for a fund with 100% market-to-book ratio due to less frequent loss occurrences. In contrast, the Swiss solvency test's 99% expected shortfall might demonstrate a non-zero capital charge.

The research highlights the vital influence of duration on tail risk. Our findings show an almost threefold increase in estimated loss frequency for a six-year fund guarantee compared to a four-year fund, with the loss size being more than twice as large. Additionally, we highlighted how the composition of the portfolio and the credit quality of assets influence tail risk. To illustrate this, we provided metrics for tail risk across various stable value fund portfolios, each with differing risk levels.

In conclusion, our research underscores the necessity for a model that can generate variable withdrawal scenarios in response to fluctuations in yield. We demonstrated that our multi-state *beta* component model outperforms deterministic trend models and rate-deficit function models in capturing the complex interplay between yield spikes, inflation, and participant withdrawal behavior. Our research contributes to a comprehensive understanding of Synthetic GIC risk management, with our model's flexibility accommodating additional factors specific to the insurance contract. This model serves as a step towards better industry practices in managing Synthetic GIC risks and a robust foundation for further research.

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Appendix A Benchmark stable value guarantee contract

Table 11 shows the most important flavors of a typical stable value guaranteed contract.

Table 11 Features of a typical guaranteed contract

Policy flavor	Value	Description
Initial market to book value	100%	
Fund duration	4 years	
Assets rating	AA	The average rating of the assets that make up the composition of the fund
Contract term	30 years	Generally, contracts are into perpetuity, but given that the Monte Carlo simulation cannot handle an infinity term, our analysis shows that—within the assumptions of our model—increasing the terms of the contract any further has an insignificant impact on the tail risk The 30 year term includes an *extended termination* period of 10 years. *Extended termination*, also known as the wind-down period, is a clause that allows the insurer to impose stronger limits on the insured assets of the portfolio. We assume that the insurer will trigger such a clause at the 20th year anniversary of the contract

Table 12 CIR estimates, used in this paper, from the log-likelihood method

	κ	ν	r_∞ or s_∞
Interest rates	0.0794	0.0656	0.0425
OAS spreads portfolio 1	0.893	0.0634	0.0070
OAS spreads portfolio 2	0.687	0.0774	0.0111
OAS spreads portfolio 3	0.661	0.0974	0.0184

Appendix B Asset model parameters assumptions

Using a log-likelihood method, we calibrated the interest rates and OAS spreads on 3-month increments. The historical calibration window spans 20 years for the Option-Adjusted Spread (OAS) spreads, sourced from [48] up to December 2021. This analysis is based on diverse asset allocations detailed in Table 6. For interest rates, a 50-year period up to September 2023 was used, sourced from 2020, focusing on the 3-year constant maturity rates. The CIR model's parameter estimation is carried out using the pre-processing and calibration method outlined in Algorithm 1 (Table 12).

Algorithm 1 Rate and Spread Calibration Methodology

1: **Input:** Daily time series of 3-year constant maturity U.S. Treasury rates or OAS spread series, noted as (t_i, r_{t_i}). Use backfilling for missing days to ensure each month has 22 days.

2: Define a new data panel of d_i, where i is the index of tge i-th business day of each quarter. A quarter has approximately 3x22 business days, hence $i \in \{1, 66\}$.

3: Reorganize the rate and spread time series into quarterly data, for each d_i, define $\left\{r_k^i\right\}$ where i is the i-th business day of the quarter, and k is the index of the quarter over the historical period analyzed. For example, if 50 years of historical data is being used, then k would be between 1 and 50x4=200 quarters; we, therefore, would have 66 times, each of size 200.

4: For each d_i, estimate the 3-month increments of $\Delta r_k^i = r_{i,k} - r_{i,k-1}$ .Apply the maximum likelihood method to Δr_k^i. Compute the log-likelihood for each of the 66 series and sum these to obtain the total likelihood. The methodology used for obtaining the log-likelihood of each of the 66 series is explained in Kladívko (2007). We then call our objective function the sum of the likelihood of all of these 66 series.

5: We fixed the θ parameter and focused on optimizing σ and κ only. For interest rates, θ was set using the mean reversion point estimated in 2020, calibrated as of June 2023. Regarding the OAS spread, we considered the average OAS spread during historical low-risk periods, excluding the financial crisis (September 2007 to December 2009) and the COVID period (February 2020 to December 2020), the 2015 stock downturn (September 2015 to October 2016), the EU sovereign crisis (January 2011 to October 2010).

6: We optimized the sum of the log-likelihood of the 66 time series to obtain the model parameters.

Appendix C Liability model parameters assumptions

Figure 12 shows function $g(\cdot)$ as an S-shape structure. The graph assumes that the rate deficit shall reach a critical level for participants to react. For example, the critical level in Fig. 12 is around $\pm 4\%$, with a maximum withdrawal rate of 20%. These numbers are just inputs and are not empirically verified. To emphasize that the function $g()$ is

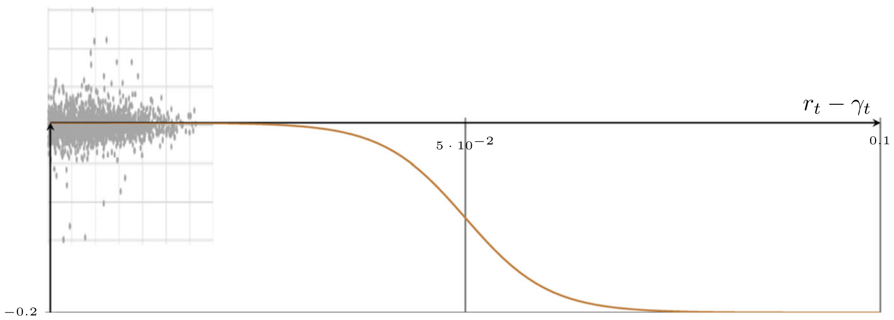

Fig. 12 Cash flow rate as a function of the crediting rate deficit. The empirical data points have been superposed to the theoretical function. Historical data source: Valerian proprietary data

Table 13 Default parameters input assumption, used in this paper, for the trend model

State	Respective trend	Probability of occurrence	Average duration (d_i)
Stable state	0	$0.85 - 0.0003 \times \mathbb{1}_{\frac{M_t}{B_t}<1.0}$	8 years
Decline state	$-1/5$	0.05	3 years
Growth state	$+1/10$	0.1	5 years
Herd state	-90%	$0.0003 \times \mathbb{1}_{\frac{M_t}{B_t}<1.0}$	0.25 years

consistent with the historical observations, Fig. 12 also overlays the graph of historical data points from Alimoradian et al. (2023) with the parametric model presented by the authors in its top-left corner.

The parametrization assumption for the rate deficit component ($g(\gamma_t - r_t)$) is as below:

$$g(x) = 0.1 \times tanh(-100x + 5)) + 0.1 \times tanh(-100x - 5) \quad \text{(C1)}$$

Therefore, the flight-to-safety component is represented by the following function:

$$f(s) = 0.2 \times \mathbb{1}_{s \geq 0.03}. \quad \text{(C2)}$$

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